EE 330
Lecture 41

Digital Circuits

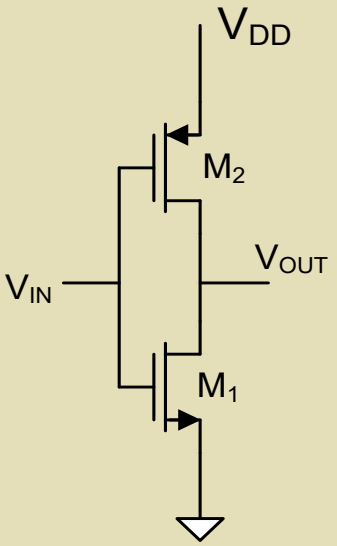
Capacitive Loading Effects on Propagation Delay
Overdrive Factors
Propagation Delay With Multiple Levels of Logic

Fall 2023 Exam Schedule

Exam 1	**Friday Sept 22**	
Exam 2	**Friday Oct 20**	
Exam 3	**Friday Nov. 17**	
Final	**Monday Dec 11**	**12:00 – 2:00 p.m.**

The Reference Inverter

Reference Inverter



Assume $\mu_n/\mu_p=3$

$W_n=W_{MIN}$, $W_p=3W_{MIN}$

$L_n=L_p=L_{MIN}$

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF, $R_{PDREF}=R_{PUREF}$=2.5K

$$R_{PDREF}=R_{PUREF}$$

$$\mathbf{C_{REF}=C_{IN}=4C_{OX}W_{MIN}L_{MIN}}$$

$$R_{PDREF}=\frac{L_{MIN}}{\mu_n C_{OX} W_{MIN}\left(V_{DD}-V_{Tn}\right)} \overset{V_{Tn}=.2V_{DD}}{=} \frac{L_{MIN}}{\mu_n C_{OX} W_{MIN}\left(0.8V_{DD}\right)}$$

$$\mathbf{t_{HLREF}=t_{LHREF}=R_{PDREF}C_{REF}}$$

$$\mathbf{t_{REF}=t_{HLREF}+t_{LHREF}=2R_{PDREF}C_{REF}}$$

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)

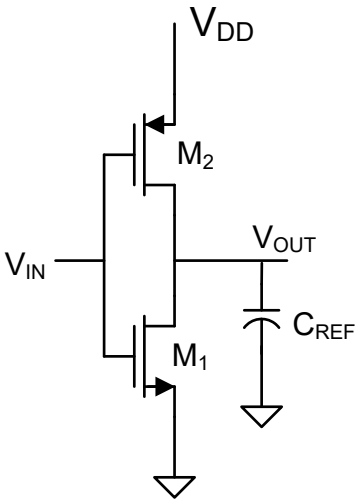
Device Sizing

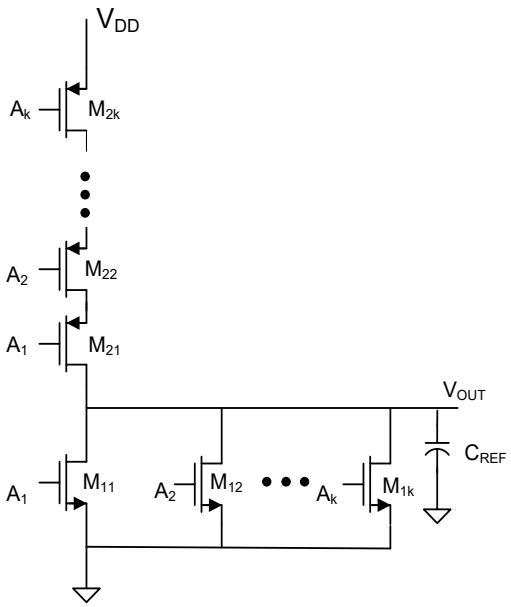
Equal Worse-Case Rise/Fall Device Sizing Strategy (and same drive as ref inverter)

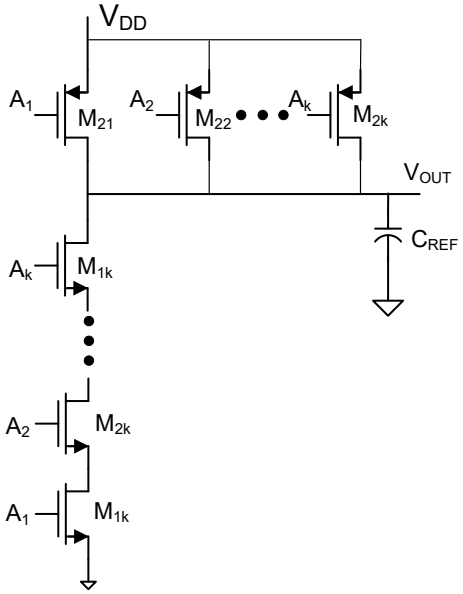
-- (same as $V_{TRIP}=V_{DD}/2$ for worst case delay in typical process considered in example)

Assume $\mu_n/\mu_p=3$ **How many degrees of freedom were available?**

$L_n=L_p=L_{MIN}$







INV

$W_n=W_{MIN}$, $W_p=3W_{MIN}$

$C_{IN}=C_{REF}$

FI=1

k-input NOR

$W_n=W_{MIN}$, $W_p=3kW_{MIN}$

$$C_{IN}=\left(\frac{3k+1}{4}\right)C_{REF}$$

$$FI=\left(\frac{3k+1}{4}\right)$$

k-input NAND

$W_n=kW_{MIN}$, $W_p=3W_{MIN}$

$$C_{IN}=\left(\frac{3+k}{4}\right)C_{REF}$$

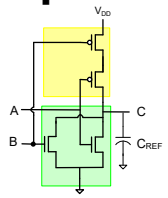
$$FI=\left(\frac{3+k}{4}\right)$$

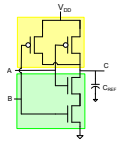
Review from Last Time

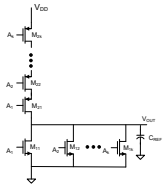
Device Sizing

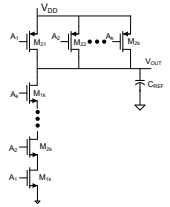
Multiple Input Gates:

2-input NOR **2-input NAND** **k-input NOR** **k-input NAND**









Equal Worst Case Rise/Fall (and equal to that of ref inverter when driving C_{REF})

Wn=?
Wp=?

Fastest response (t_{HL} or t_{LH}) = ?

Worst case response (t_{PROP}, usually of most interest)?

Input capacitance (FI) = ?

Minimum Sized (assume driving a load of C_{REF})

Wn=Wmin
Wp=Wmin

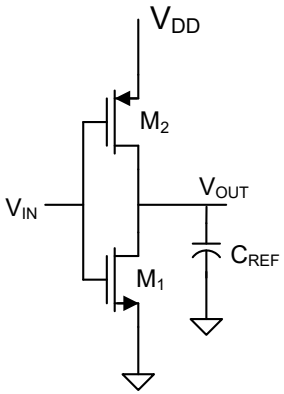
Fastest response (t_{HL} or t_{LH}) = ?

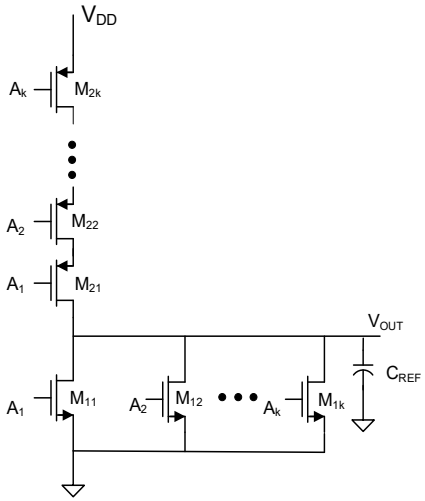
Slowest response (t_{HL} or t_{LH}) = ?

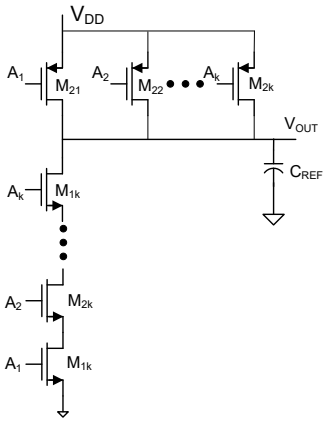
Worst case response (t_{PROP}, usually of most interest)?

Input capacitance (FI) = ?

Device Sizing – minimum size driving C_{REF}







INV

worst case

$$t_{PROP} = 0.5t_{REF} + \frac{3}{2}t_{REF}$$

$$t_{PROP} = 2t_{REF}$$

$$FI = \frac{C_{REF}}{2}$$

$$R_{PU} = 3R_{PDREF}$$

$$R_{PD} = R_{PDREF}$$

k-input NOR

$$t_{PROP} = 0.5t_{REF} + \frac{3k}{2}t_{REF}$$

$$t_{PROP} = \left(\frac{3k+1}{2}\right)t_{REF}$$

$$\frac{1+3k^2}{2k}t_{REF} \le t_{PROP} \le \frac{3k+1}{2}t_{REF}$$

$$FI = \frac{C_{REF}}{2}$$

$$\frac{R_{PDREF}}{k} \le R_{PD} \le R_{PDREF}$$

$$R_{PU} = 3kR_{PDREF}$$

k-input NAND

$$t_{PROP} = \frac{3}{2}t_{REF} + \frac{k}{2}t_{REF}$$

$$t_{PROP} = \frac{3+k}{2}t_{REF}$$

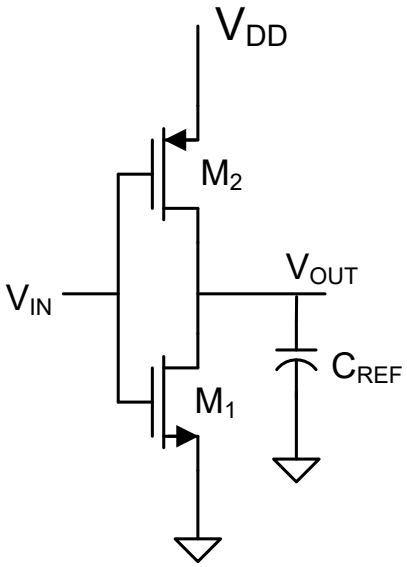
$$\frac{3+k^2}{2k}t_{REF} \le t_{PROP} \le \frac{3+k}{2}t_{REF}$$

$$FI = \frac{C_{REF}}{2}$$

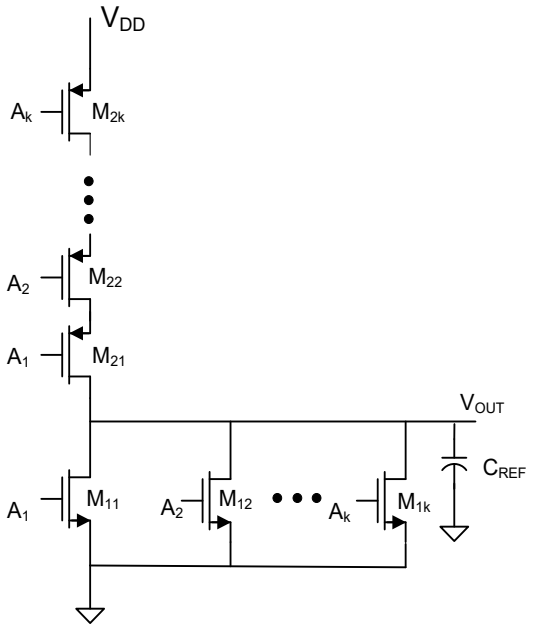
$$\frac{3R_{PDREF}}{k} \le R_{PU} \le 3R_{PDREF}$$

$$R_{PD} = kR_{PDREF}$$

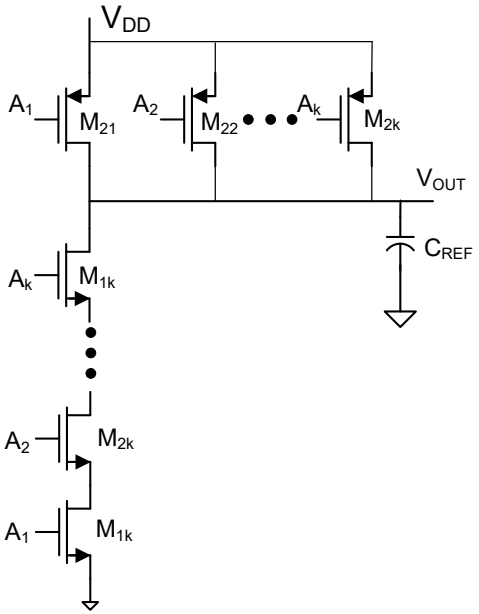
Device Sizing Summary



INV



k-input NOR



k-input NAND

C_{IN} for N_{AND} gates is considerably smaller than for NOR gates for equal worst-case rise and fall times

C_{IN} for minimuim-sized structures is independent of number of inputs and much smaller than C_{IN} for the equal rise/fall time case

R_{PU} gets very large for minimum-sized NOR gate

Digital Circuit Design

- Hierarchical Design
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter
- Static CMOS Logic Gates
  - Ratio Logic
- Propagation Delay
  - Simple analytical models
    - FI/OD
    - Logical Effort
  - Elmore Delay
- Sizing of Gates
  - The Reference Inverter

- Propagation Delay with Multiple Levels of Logic
- Optimal driving of Large Capacitive Loads
- Power Dissipation in Logic Circuits
- Other Logic Styles
- Array Logic
- Ring Oscillators

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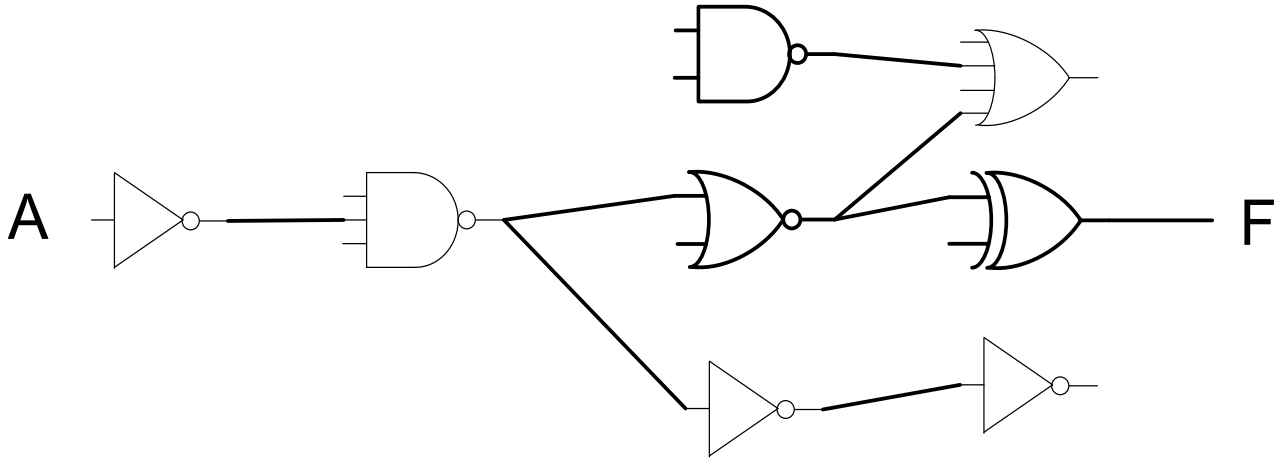
done

partial

Propagation Delay in Multiple-Levels of Logic with Stage Loading



Assume all gates sized for equal worst-case rise/fall times

For n levels of logic between A and F

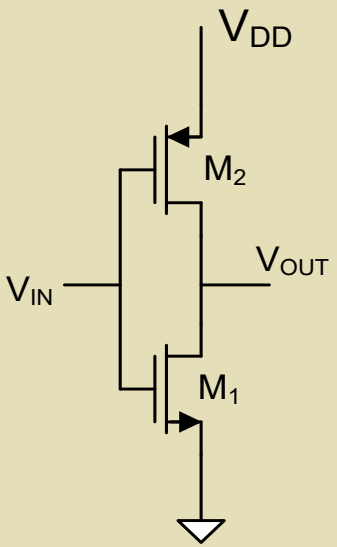
$$t_{PROP} = \sum_{k=1}^{n} t_{PROP}(k)$$

Remember: t_{prop} is defined to be the worst-case (slowest) propagation delay

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Analysis strategy : Express delays in terms of those of reference inverter

Reference Inverter



Assume $\mu_n/\mu_p=3$

$W_n=W_{MIN}$, $W_p=3W_{MIN}$

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF, R_{PDREF}=2.5K

$$C_{REF}=C_{IN}=4C_{OX}W_{MIN}L_{MIN}$$

FI= 1

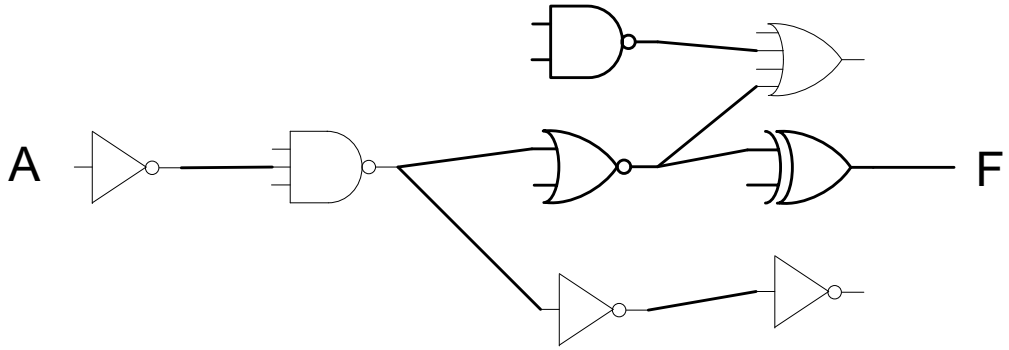
$$R_{PDREF}=\frac{L_{MIN}}{\mu_n C_{OX} W_{MIN}\left(V_{DD}-V_{Tn}\right)} \overset{V_{Tn}=.2V_{DD}}{=} \frac{L_{MIN}}{\mu_n C_{OX} W_{MIN}\left(0.8V_{DD}\right)}$$

$$t_{REF}=t_{HLREF}+t_{LHREF}=2R_{PDREF}C_{REF}$$

$$L_n=L_p=L_{MIN}$$

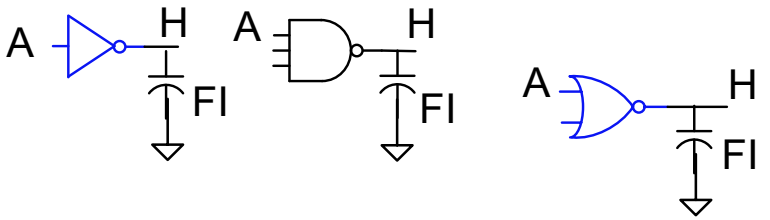
(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)

Propagation Delay in Multiple-Levels of Logic with Stage Loading



Assume:

- **all gates sized for equal worst-case rise/fall times**
- **all gates sized to have rise and fall times equal to that of ref inverter when driving C_{REF}**

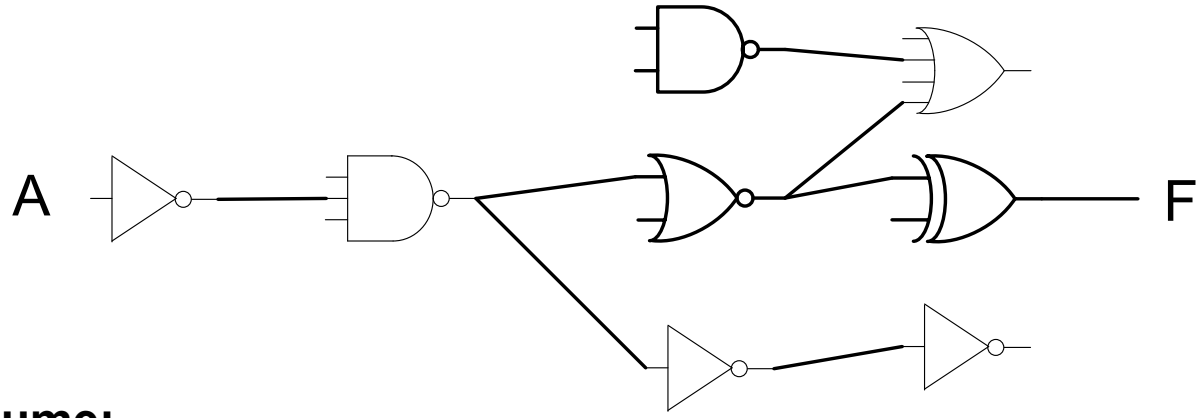


$t_{PROP-AH}=t_{ref}Fl$

Observe:

- **With these assumptions, propagation delay of these gates will be scaled by the ratio of the total load capacitance on each gate to C_{REF}**

Propagation Delay in Multiple-Levels of Logic with Stage Loading



Assume:

- **all gates sized for equal worst-case rise/fall times**
- **all gates sized to have rise and fall times equal to that of ref inverter when driving C_{REF}**

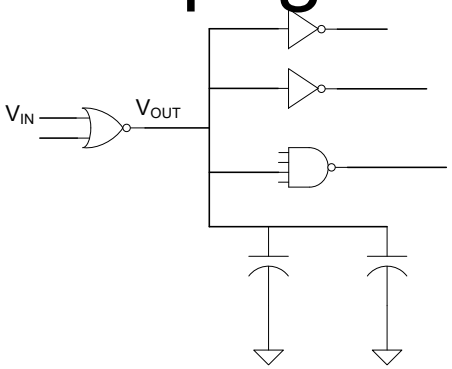
Observe:

- **Propagation delay of these gates will be scaled by the ratio of the total load capacitance on each gate to C_{REF}**

What loading will a gate see?

- **Input capacitance to other gates**
- **Any load capacitors**
- **Parasitic interconnect capacitnaces**

Propagation Delay with Stage Loading



$$t_{REF} = 2R_{PDref}C_{REF}$$

$$C_{REF} = 4C_{OX}W_{MIN}L_{MIN}$$

FI of a capacitor

$$FI_C = \frac{C}{C_{REF}}$$

FI of a gate (input k)

$$FI_G = \frac{C_{INk}}{C_{REF}}$$

FI of an interconnect

$$FI_I = \frac{C_{INI}}{C_{REF}}$$

Overall FI

$$FI = \frac{\sum_{\text{Gates}} C_{INGi} + \sum_{\text{Capacitances}} C_{INCi} + \sum_{\text{Interconnects}} C_{INIi}}{C_{REF}}$$

FI can be expressed either in units of capacitance or normalized to C_{REF}

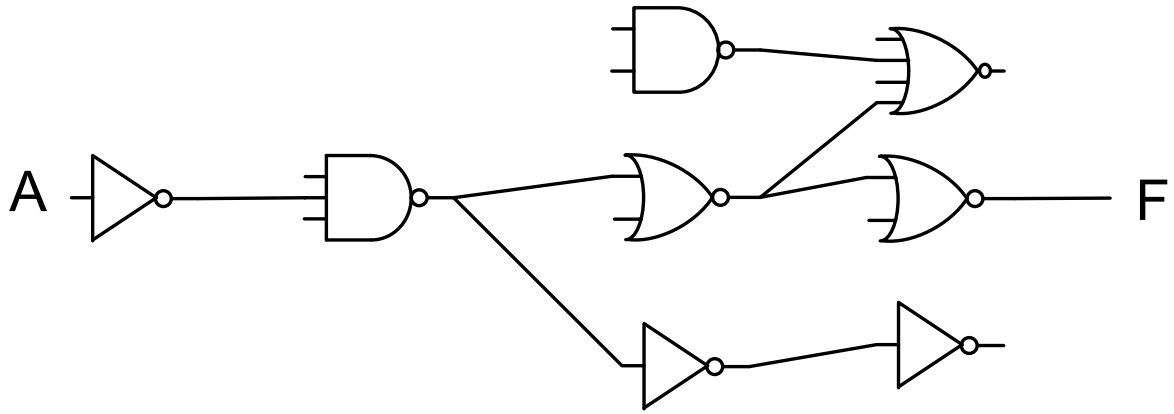
Most commonly FI is normalized but must determine from context

If gates sized to have same drive as ref inverter

$$t_{prop\text{-}k} = t_{REF} \bullet FI_{LOAD\text{-}k}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Example



Assume all gates sized for equal worst-case rise/fall times

Assume all gate drives are the same as that of reference inverter

Neglect interconnect capacitance, assume load of $10C_{REF}$ on F output

Determine propagation delay from A to F

$$t_{PROP} = \sum_{k=1}^{4} t_{PROP\text{-}k}$$

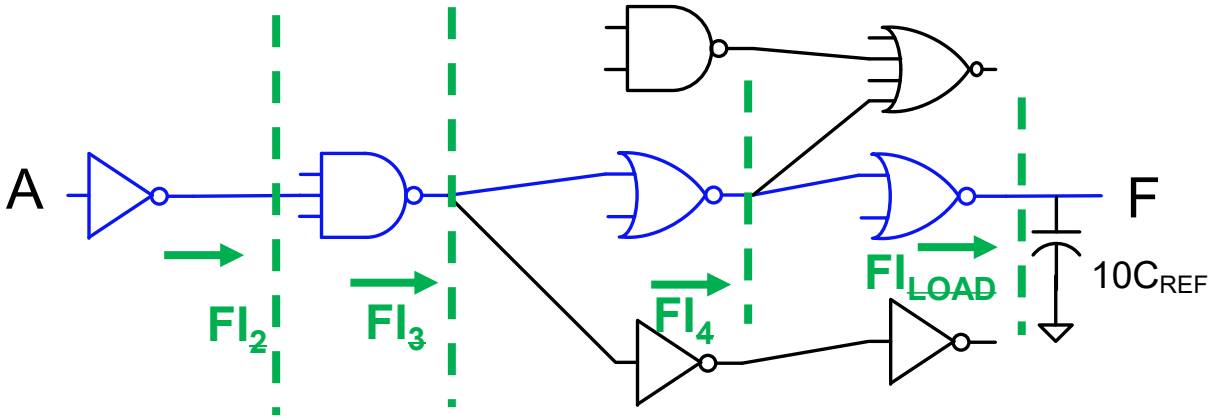
$$t_{prop\text{-}k} = t_{REF} \bullet FI_{LOAD\text{-}k}$$

$$t_{PROP} = t_{REF} \sum_{k=1}^{4} FI_{LOAD\text{-}k}$$

$$FI_{LOAD\text{-}k} = FI_{k+1}$$

$$t_{PROP} = t_{REF} \sum_{k=1}^{4} FI_{k+1}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading



$$FI_{NOR} = \left(\frac{3k+1}{4}\right) C_{REF}$$

$$FI_{NAND} = \left(\frac{3+k}{4}\right) C_{REF}$$

Assume all gates sized for equal worst-case rise/fall times

Assume all gate drives are the same as that of reference inverter

Neglect interconnect capacitance, assume load of $10C_{REF}$ on F output

Determine propagation delay from A to F

$$t_{PROP} = t_{REF} \sum_{k=1}^{4} FI_{k+1}$$

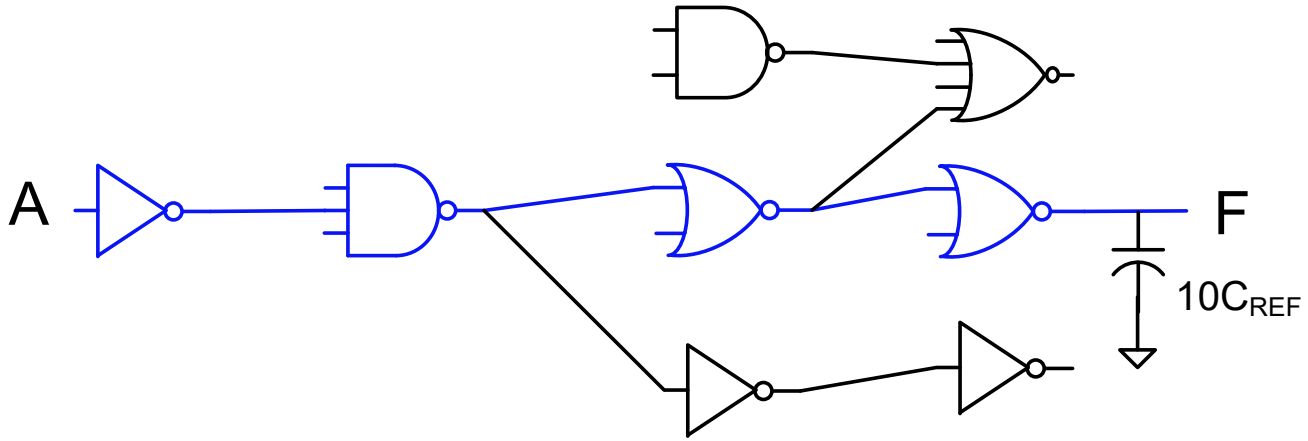
What loading will a gate see?

Derivation:

$$FI_2 = \frac{6}{4} C_{REF} \qquad FI_3 = C_{REF} + \frac{7}{4} C_{REF} \qquad FI_4 = \frac{7}{4} C_{REF} + \frac{13}{4} C_{REF} \qquad FI_{LOAD} = FI_{"5"} = 10C_{REF}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Example



Assume all gates sized for equal worst-case rise/fall times

Assume all gate drives are the same as that of reference inverter

Neglect interconnect capacitance, assume load of $10C_{REF}$ on F output

Determine propagation delay from A to F

$$t_{PROP} = t_{REF} \sum_{k=1}^{4} FI_{k+1}$$

DERIVATIONS

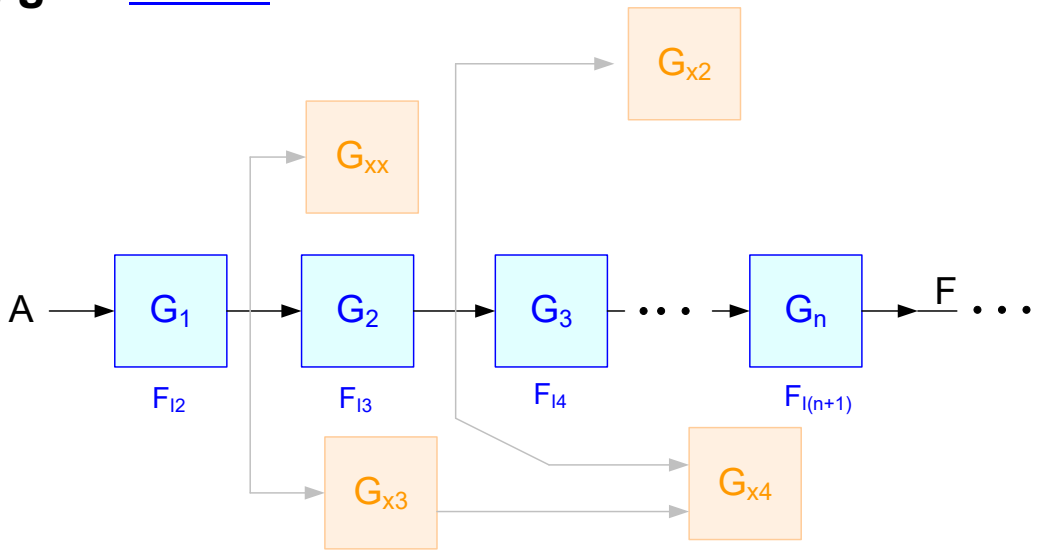
$$FI_2 = \frac{6}{4} C_{REF} \qquad FI_3 = C_{REF} + \frac{7}{4} C_{REF} \qquad FI_4 = \frac{7}{4} C_{REF} + \frac{13}{4} C_{REF} \qquad FI_5 = 10C_{REF}$$

$$t_{PROP1} = \frac{6}{4} t_{REF} \qquad t_{PROP2} = \left(1 + \frac{7}{4}\right) t_{REF} \qquad t_{PROP3} = \left(\frac{7}{4} + \frac{13}{4}\right) t_{REF} \qquad t_{PROP4} = 10 t_{REF}$$

$$t_{PROP} = \sum_{k=1}^{n} t_{PROP\text{-}k} = t_{REF} \sum_{k=1}^{n} FI_{k+1} = t_{REF} \left(\frac{6}{4} + \frac{11}{4} + \frac{20}{4} + 10\right) = t_{REF}\,(19.25)$$

Propagation Delay Through Multiple Stages of Logic with Stage Loading

(assuming gate drives are all same as that of reference inverter)



Summary:

Identify the gate path from A to F

$$t_{PROPk} = t_{REF} FI_{k+1}$$

Propagation delay from A to F:

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{k+1}$$

This approach is analytically manageable, provides modest accuracy and is "faithful"

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- Optimal driving of Large Capacitive Loads
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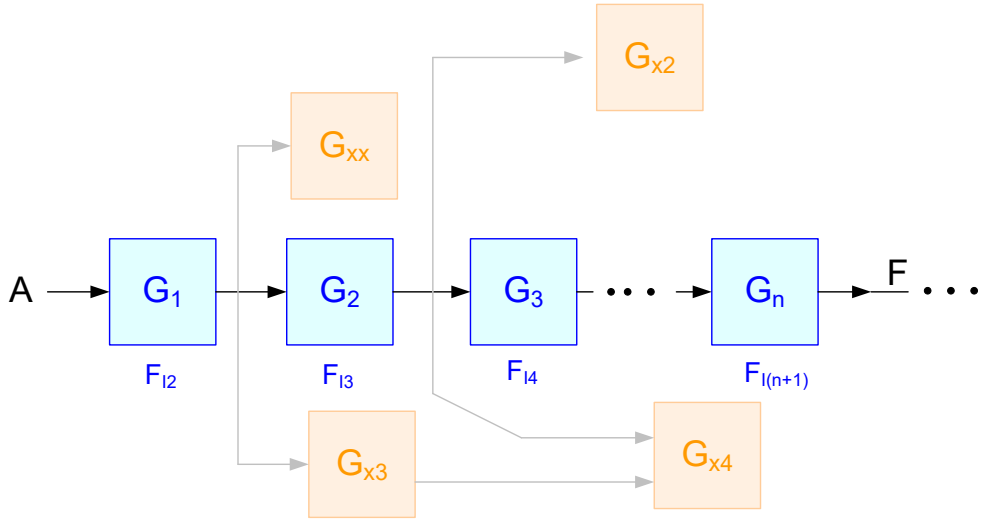
done

partial

What if the propagation delay is too long (or too short)?



Propagation delay from A to F:

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

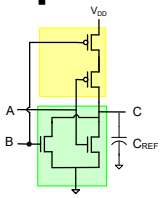
$$\mathbf{t_{PROPk} = t_{REF} FI_{(k+1)}}$$

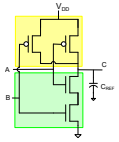
Recall:

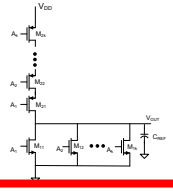
Device Sizing

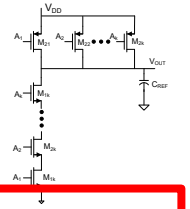
Multiple Input Gates:

2-input NOR **2-input NAND** **k-input NOR** **k-input NAND**









Equal Worst Case Rise/Fall **(and equal to that of ref inverter when driving C_{REF})**

consider the fine print !

W_n=?

W_p=?

Fastest response (t_{HL} or t_{LH}) = ?

Worst case response (t_{PROP}, usually of most interest)?

Input capacitance (FI) = ?

Minimum Sized **(assume driving a load of C_{REF})**

$W_n=W_{min}$

$W_p=W_{min}$

Fastest response (t_{HL} or t_{LH}) = ?

Slowest response (t_{HL} or t_{LH}) = ?

Worst case response (t_{PROP}, usually of most interest)?

Input capacitance (FI) = ?

Recall:

Device Sizing

Equal Worst Case Rise/Fall

(and equal to that of ref inverter when driving C_{REF})

Multiple Input Gates: 2-input NOR

(n-channel devices sized same, p-channel devices sized the same)

Assume $L_n=L_p=L_{min}$ and driving a load of C_{REF}

W_n=?

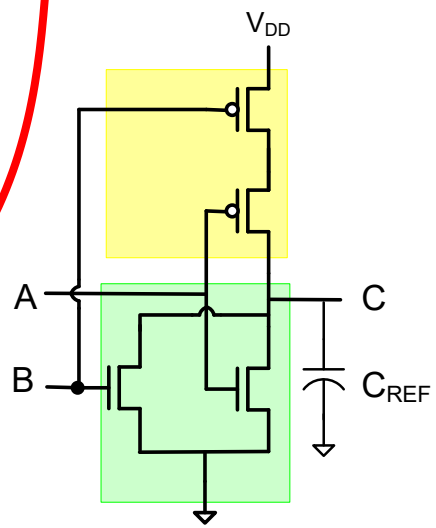
DERIVATIONS

W_p=?

Input capacitance = ?

FI=?

t_{PROP}=? (worst case)



One degree of freedom was used to satisfy the constraint indicated

$W_n=W_{MIN}$

$W_p=6W_{MIN}$

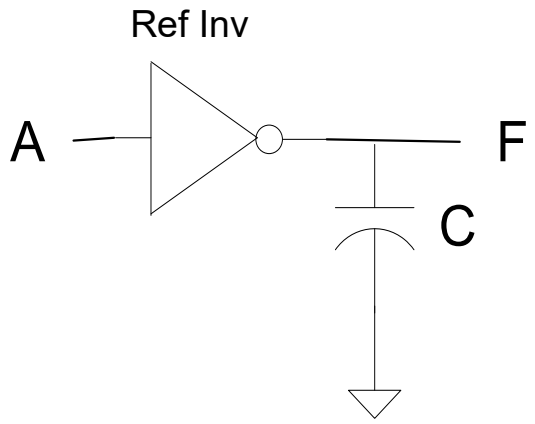
Other degree of freedom was used to achieve equal rise and fall times

$$C_{INA}=C_{INB}=C_{OX}W_{MIN}L_{MIN}+6C_{OX}W_{MIN}L_{MIN}=7C_{OX}W_{MIN}L_{MIN}=\left(\frac{7}{4}\right)4C_{OX}W_{MIN}L_{MIN}=\left(\frac{7}{4}\right)C_{REF}$$

$$FI=\left(\frac{7}{4}\right)C_{REF} \quad or \quad FI=\frac{7}{4}$$

$t_{PROP} = t_{REF}$ (worst case)

Overdrive Factors



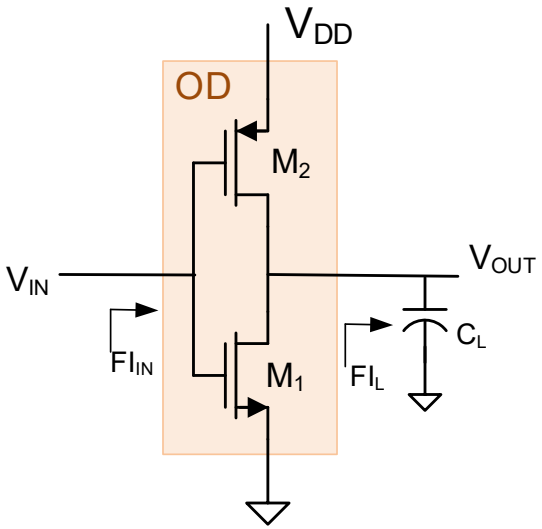
Example: Determine t_{prop} in 0.5u process if C=10pF In 0.5u proc t_{REF}=20ps, C_{REF}=4fF, R_{PDREF}=2.5K

$$t_{PROP} = t_{REF} \bullet FI = t_{REF} \bullet \frac{10\,pF}{4\,fF} = t_{REF} \bullet 2500$$

$$t_{PROP} = 20ps \bullet 2500 = 50nsec$$

Note this is generally considered to be unacceptably long !

Overdrive Factors



Scaling widths of ALL devices by constant (W_{scaled}=WxOD) will change "drive" capability relative to that of the reference inverter but not change relative value of t_{HL} and t_{LH}

$$R_{PD}=\frac{L_1}{\mu_n C_{OX} W_1 (V_{DD}-V_{Tn})} \rightarrow R_{PDOD}=\frac{L_1}{\mu_n C_{OX}[OD \bullet W_1](V_{DD}-V_{Tn})}=\frac{R_{PD}}{OD}$$

$$R_{PU}=\frac{L_2}{\mu_p C_{OX} W_2 (V_{DD}+V_{Tp})} \rightarrow R_{PUOD}=\frac{L_2}{\mu_p C_{OX}[OD \bullet W_2](V_{DD}+V_{Tp})}=\frac{R_{PU}}{OD}$$

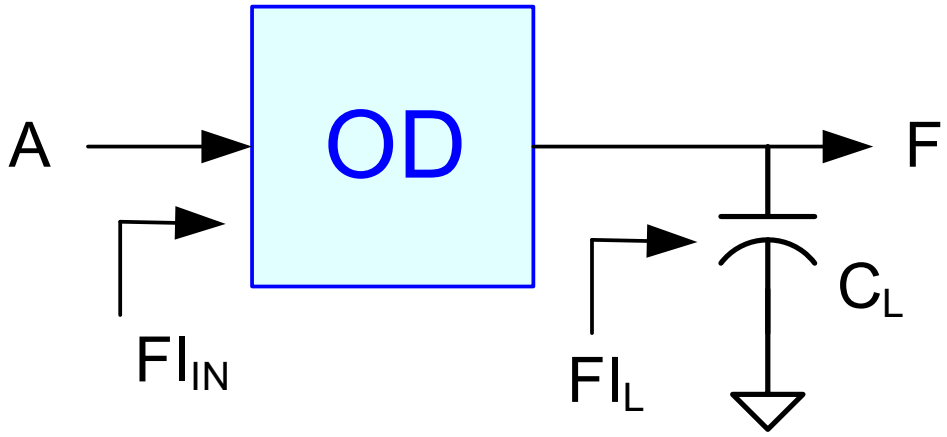
$$t_{PROP}=t_{REF} \bullet FI_L \bullet \frac{1}{OD}$$

Scaling widths of ALL devices by constant will change FI_{IN} to gate by OD

$$C_{IN}=C_{OX}(W_1L_1+W_2L_2) \rightarrow C_{INOD}=C_{OX}([OD \bullet W_1]L_1+[OD \bullet W_2]L_2)=OD \bullet C_{IN}$$

Overdrive Factors - Summary

(For equal worst-case rise/fall gates)



$$W = W_{REF} \bullet OD$$

$$R_{PD\text{-}OD} = \frac{R_{PD\text{-}REF}}{OD}$$

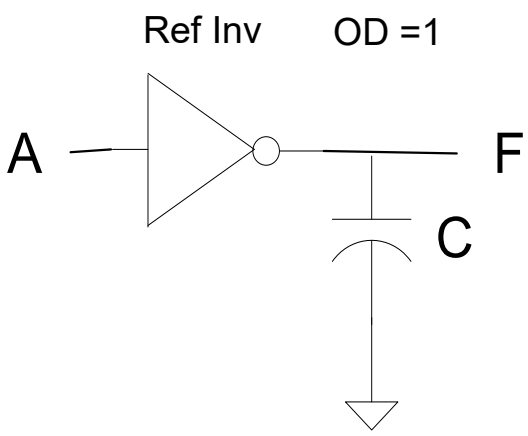
$$R_{PU\text{-}OD} = \frac{R_{PU\text{-}REF}}{OD}$$

Still equal worst-case rise/fall

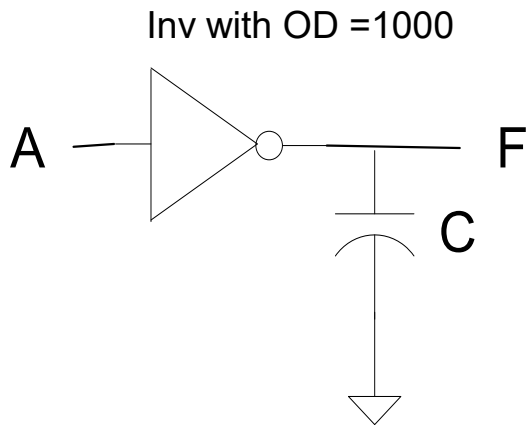
$$\mathbf{t_{PROP} = t_{REF} \bullet FI_L \bullet \frac{1}{OD}}$$

$$FI_{IN} = = OD \bullet C_{REF}$$

Overdrive Factors



t_{PROP1}=50nsec



t_{PROP2}=?

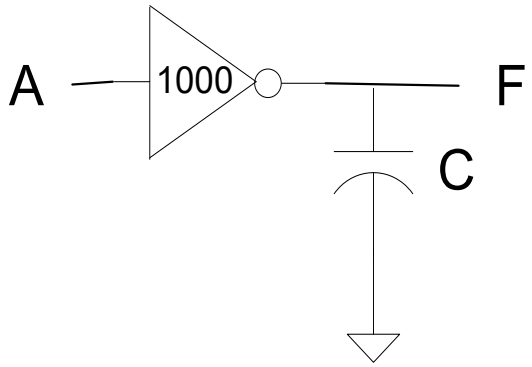
Example: Determine t_{prop} in 0.5u process if C=10pF and OD=1000

$$t_{PROP1} = t_{REF} \bullet FI_{LOAD} \bullet \frac{1}{OD} = t_{REF} \bullet \frac{10\,pF}{4\,fF} = t_{REF} \bullet 2500$$

$$t_{PROP2} = t_{REF} \bullet FI_{LOAD} \bullet \frac{1}{OD} = t_{REF} \bullet \frac{10\,pF}{4\,fF} \bullet \frac{1}{1000} = t_{REF} \bullet 2.5$$

Note sizing the inverter with the OD improved delay by a factor of 1000 !

Overdrive Factors



- **By definition, the factor by which the W/L of all devices are scaled above those of the reference inverter is termed the overdrive factor, OD**
- **Scaling widths by overdrive factor DECREASES resistance by same factor**
- **Scaling <u>all</u> widths by a constant does not compromise the symmetry between the rise and fall times (i.e. $t_{HL}=t_{LH}$)**
- **Judicious use of overdrive can dramatically improve the speed of digital circuits**
- **Large overdrive factors are often used**
- **Scaling widths by overdrive factor INCREASES input capacitance by same factor - So is there any net gain in speed?**

Digital Circuit Design

- Hierarchical Design
- Basic Logic Gates
- Properties of Logic Families
- Characterization of CMOS Inverter
- Static CMOS Logic Gates
  - Ratio Logic
- Propagation Delay
  - Simple analytical models
    - FI/OD
    - Logical Effort
  - Elmore Delay
- Sizing of Gates
  - The Reference Inverter

- Propagation Delay with Multiple Levels of Logic
- Optimal driving of Large Capacitive Loads
- Power Dissipation in Logic Circuits
- Other Logic Styles
- Array Logic
- Ring Oscillators

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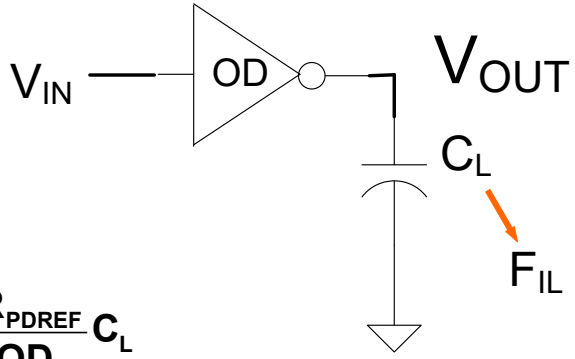
done

partial

Propagation Delay with Over-drive Capability

Overdrive



$$t_{HL}=t_{LH}=\frac{R_{PDREF}}{OD}C_L$$

$$t_{PROP}=t_{HL}+t_{LH}=2\frac{R_{PDREF}C_L}{OD}=\frac{t_{REF}}{OD}$$

Asymmetric Overdrive

Define the Asymmetric Overdrive Factors of the stage to be the factor by which PU and PD resistors are scaled relative to those of the reference inverter.

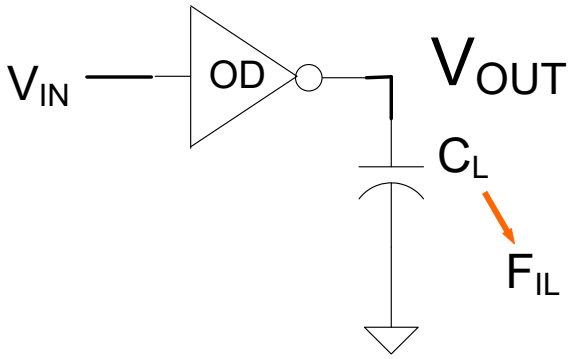
$$R_{PDEFF}=\frac{R_{PDREF}}{OD_{HL}} \qquad R_{PUEFF}=\frac{R_{PUREF}}{OD_{LH}}$$

$$t_{HL}=\frac{R_{PDREF}}{OD_{HL}}C_L \qquad t_{LH}=\frac{R_{PDREF}}{OD_{LH}}C_L$$

$$t_{PROP}=t_{HL}+t_{LH}=\frac{R_{PDREF}}{OD_{HL}}C_L+\frac{R_{PDREF}}{OD_{LH}}C_L=R_{PDREF}C_L\left[\frac{1}{OD_{HL}}+\frac{1}{OD_{LH}}\right]=\frac{t_{REF}}{2}\left[\frac{1}{OD_{HL}}+\frac{1}{OD_{LH}}\right]F_{IL}$$

Propagation Delay with Over-drive Capability

Overdrive



If an inverter with OD is sized for equal rise/fall, $OD_{HL}=OD_{LH}=OD$

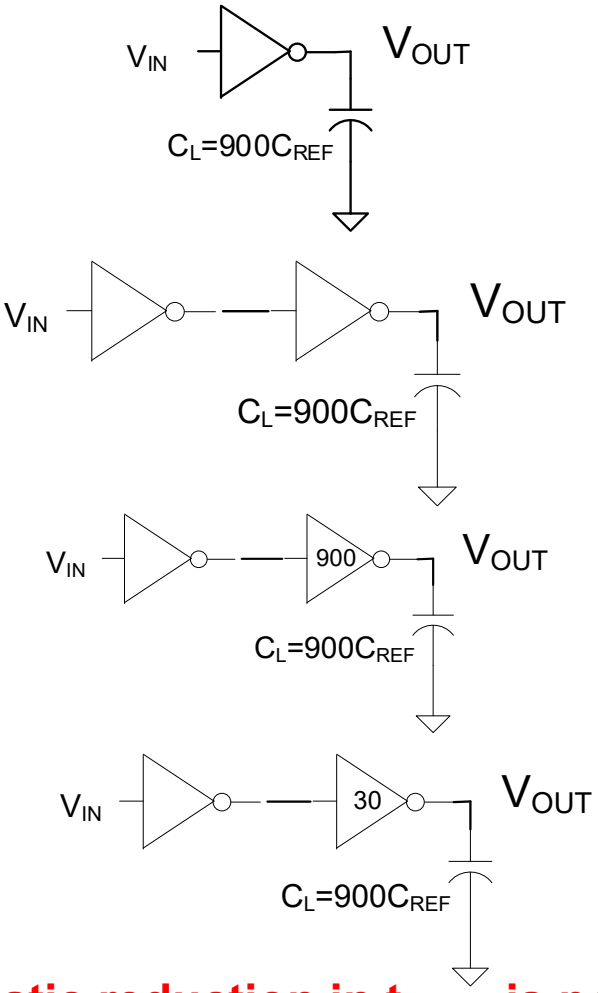
$$t_{PROP}=R_{PDREF}C_L\left[\frac{1}{OD_{HL}}+\frac{1}{OD_{LH}}\right]=R_{PDREF}C_L\frac{2}{OD}=t_{REF}\frac{F_{IL}}{OD}$$

OD may be larger or smaller than 1

Propagation Delay with Over-drive Capability

Example

Compare the propagation delays. Assume the OD is 900 in the third case and 30 in the fourth case. Don't worry about the extra inversion at this time.



$$t_{PROP}=900t_{REF}$$

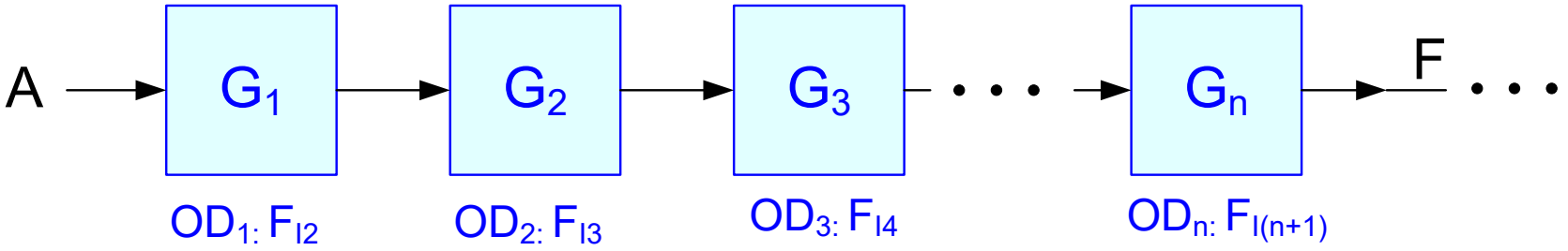
$$t_{PROP}=t_{REF}+900t_{REF}=901t_{REF}$$

$$t_{PROP}=900t_{REF}+t_{REF}=901t_{REF}$$

$$t_{PROP}=30t_{REF}+30t_{REF}=60t_{REF}$$

- **Dramatic reduction in t_{PROP} is possible** (input is driving same in all 3 cases)
- **Will later determine what optimal number of stages and sizing is**

Propagation Delay in Multiple-Levels of Logic with Stage Loading



F_{Ik} denotes the total loading on stage k which is the sum of the F_I of all loading on stage k

Summary: Propagation delay from A to F:

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$$

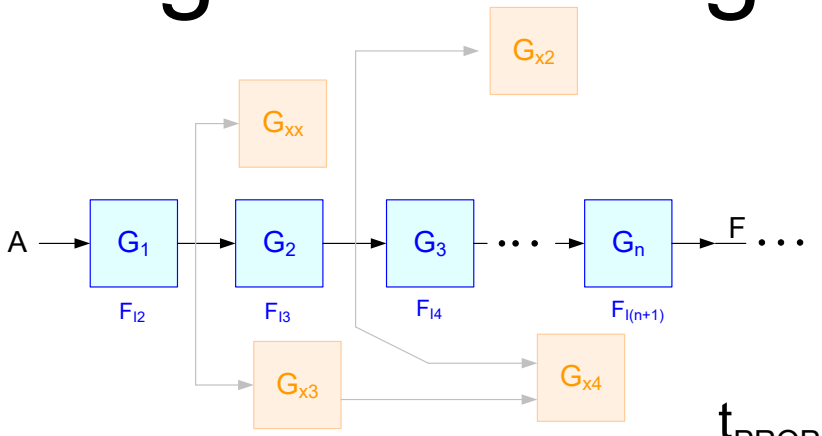
Propagation Delay in Multiple-Levels of Logic with Stage Loading

Will consider an example with the five cases

- **Equal rise/fall (no overdrive)**
- **Equal rise/fall with overdrive**
- **Minimum Sized**
- **Asymmetric Overdrive**
- **Combination of equal rise/fall, minimum size and overdrive**

Will develop the analysis methods as needed

Propagation Delay in Multiple-Levels of Logic with Stage Loading



- **Equal rise/fall (no overdrive)**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

- **Equal rise/fall with overdrive**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$$

- **Minimum Sized**

$$t_{PROP} = \ ?$$

- **Asymmetric Overdrive**

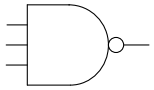
$$t_{PROP} = \ ?$$

- **Combination of equal rise/fall, minimum size and overdrive**

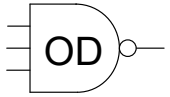
$$t_{PROP} = \ ?$$

Driving Notation

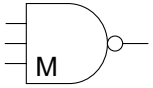
- **Equal rise/fall (no overdrive)**



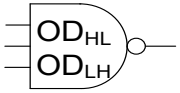
- **Equal rise/fall with overdrive**



- **Minimum Sized**



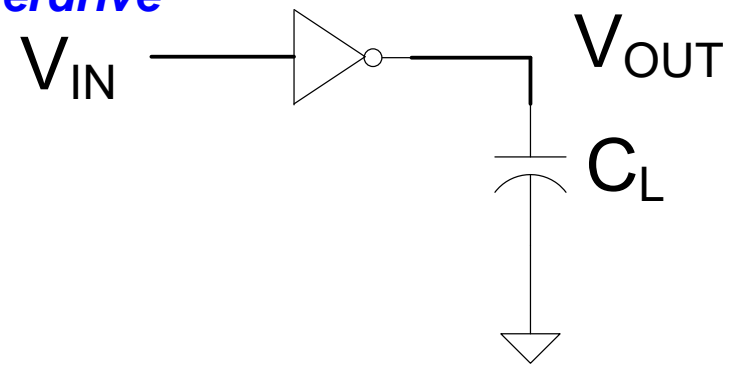
- **Asymmetric Overdrive**



Notation will be used only if it is not clear from the context what sizing is being used

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Asymmetric Overdrive



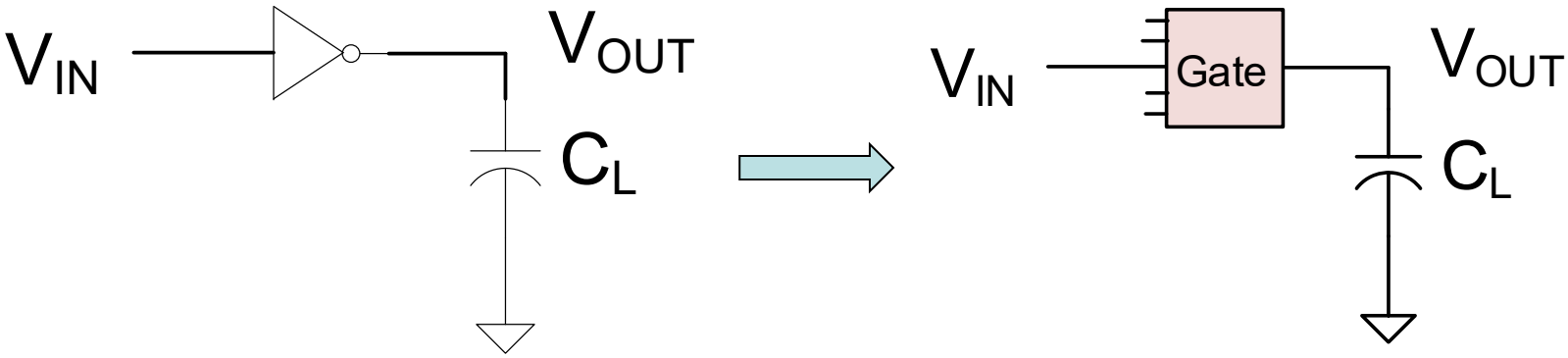
Recall:

Define the Asymmetric Overdrive Factors of the stage to be the factors by which PU and PD resistors are scaled relative to those of the reference inverter.

$$R_{PDEFF} = \frac{R_{PDREF}}{OD_{HL}} \qquad R_{PUEFF} = \frac{R_{PUREF}}{OD_{LH}}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Asymmetric Overdrive



Recall:

$$t_{PROP} = t_{LH} + t_{HL} = t_{REF} \frac{F_{IL}}{OD}$$

If inverter is not equal rise/fall

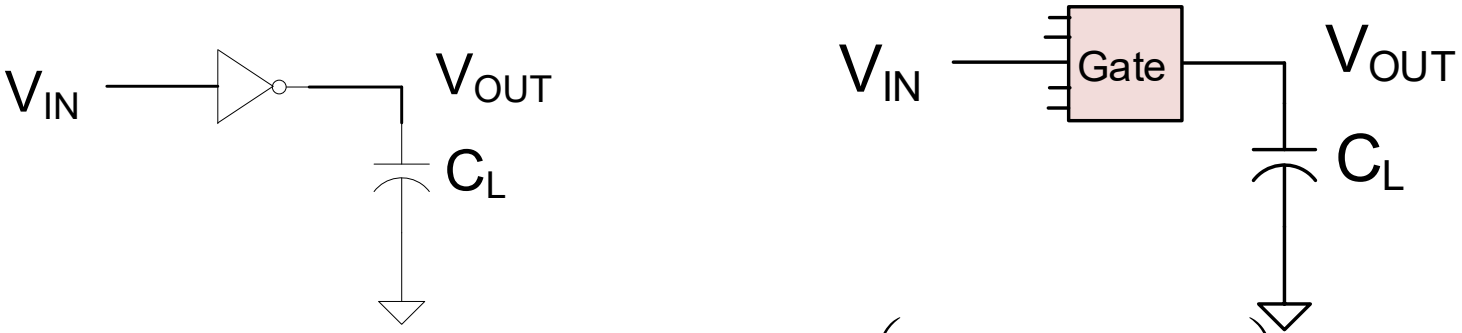
$$t_{HL} = \frac{R_{PDREF}}{OD_{HL}} C_L = \frac{1}{2} t_{REF} \frac{F_{IL}}{OD_{HL}}$$

$$t_{LH} = \frac{R_{PUREF}}{OD_{LH}} C_L = \frac{1}{2} t_{REF} \frac{F_{IL}}{OD_{LH}}$$

$$t_{PROP} = t_{HL} + t_{LH} = \frac{1}{2} t_{REF} F_{IL} \left(\frac{1}{OD_{HL}} + \frac{1}{OD_{LH}} \right)$$

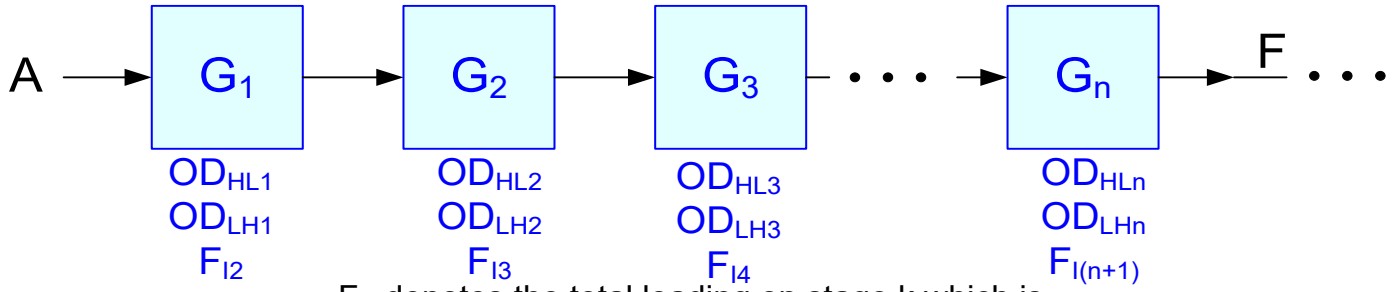
Propagation Delay in Multiple-Levels of Logic with Stage Loading

Asymmetric Overdrive



$$t_{PROP} = t_{HL} + t_{LH} = \frac{1}{2} t_{REF} F_{ILOAD} \left(\frac{1}{OD_{HL}} + \frac{1}{OD_{LH}} \right)$$

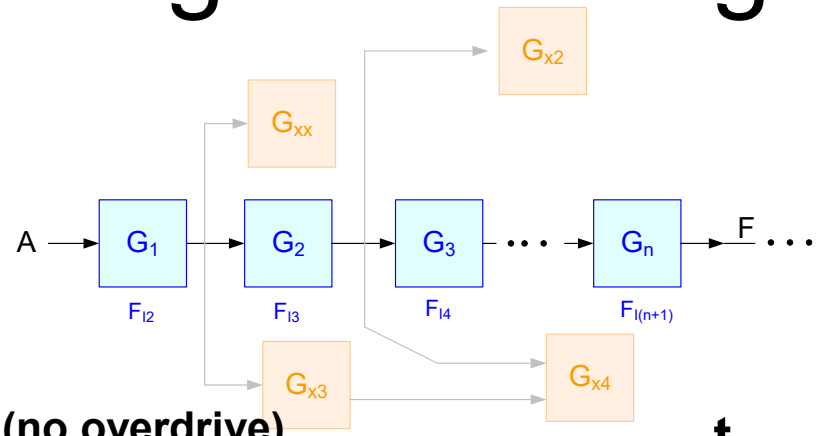
When propagating through n stages:



F_{Ik} denotes the total loading on stage k which is the sum of the F_I of all loading on stage k

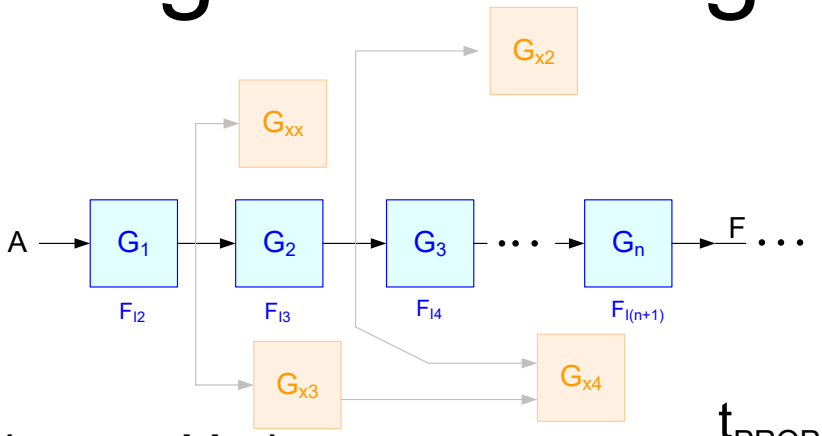
$$t_{PROP} = t_{REF} \bullet \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading



- **Equal rise/fall (no overdrive)** $t_{PROP} = ?$
- **Equal rise/fall with overdrive** $t_{PROP} = ?$
- **Minimum Sized** $t_{PROP} = ?$
- **Asymmetric Overdrive** $t_{PROP} = ?$
- **Combination of equal rise/fall, minimum size and overdrive** $t_{PROP} = ?$

Propagation Delay in Multiple-Levels of Logic with Stage Loading



- **Equal rise/fall (no overdrive)**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

- **Equal rise/fall with overdrive**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$$

- **Minimum Sized**

$$t_{PROP} = \ ?$$

- **Asymmetric Overdrive**

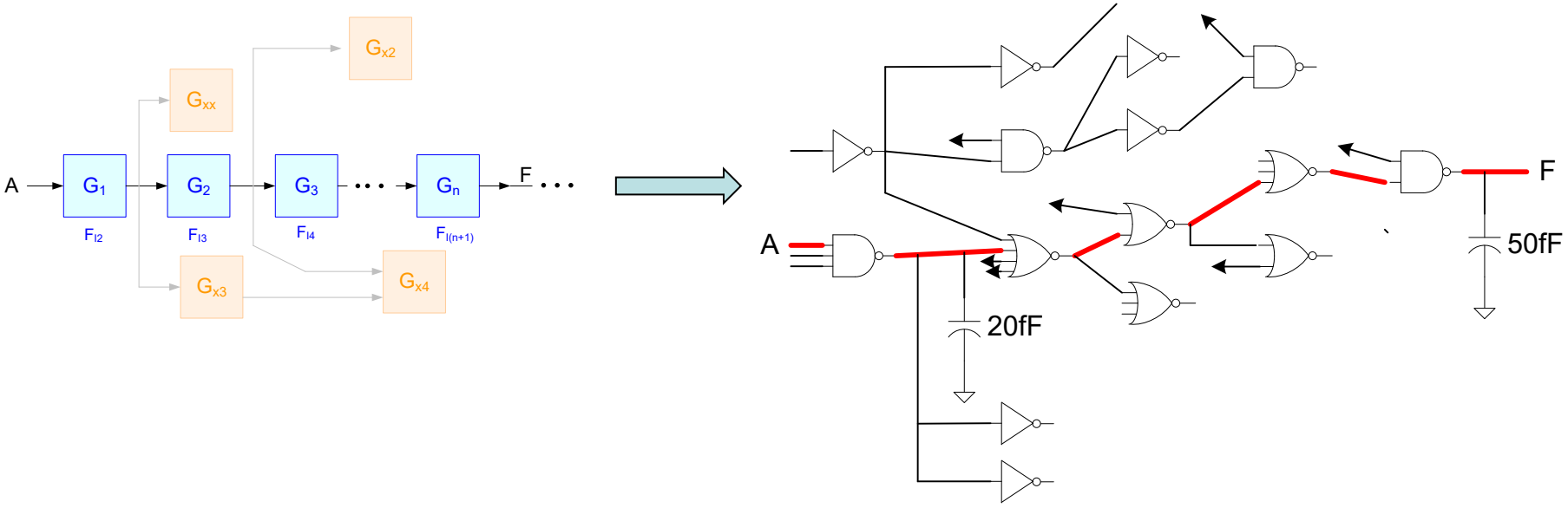
$$t_{PROP} = t_{REF} \bullet \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

- **Combination of equal rise/fall, minimum size and overdrive**

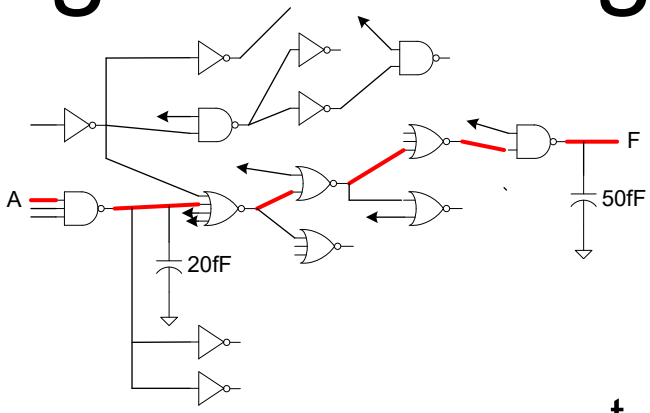
$$t_{PROP} = \ ?$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading and Overdrives

Will now consider A to F propagation for this circuit as an example with different overdrives



Propagation Delay in Multiple-Levels of Logic with Stage Loading



- **Equal rise/fall (no overdrive)**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

- **Equal rise/fall with overdrive**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$$

- **Minimum Sized**

$$t_{PROP} = \ ?$$

- **Asymmetric Overdrive**

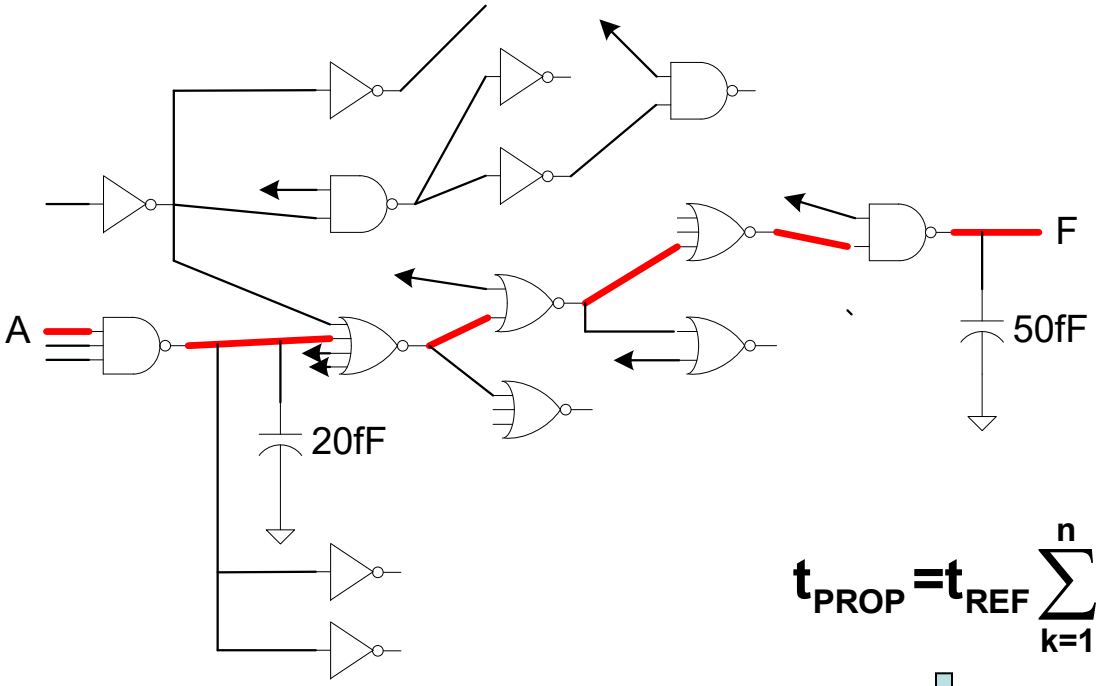
$$t_{PROP} = t_{REF} \bullet \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

- **Combination of equal rise/fall, minimum size and overdrive**

$$t_{PROP} = \ ?$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Equal rise-fall gates, no overdrive



$$t_{PROP} = t_{REF} \sum_{k=1}^{n} F_{I_{k+1}}$$

$$\Downarrow$$

$$\frac{t_{PROP}}{t_{REF}} = \sum_{k=1}^{n} F_{I_{k+1}}$$

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

Propagation Delay in Multiple-Levels of Logic with Stage Loading

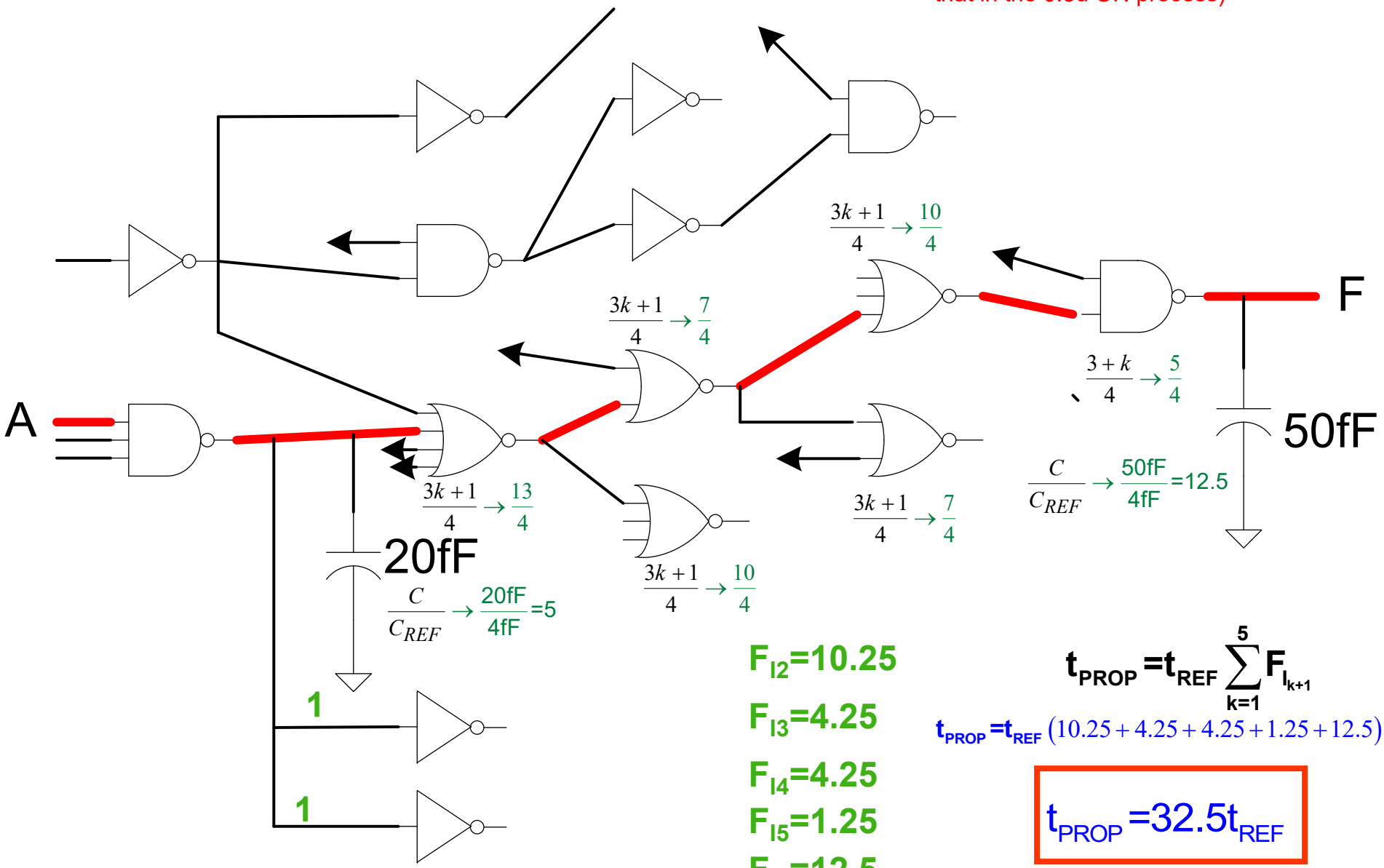
Equal rise-fall gates, no overdrive

	Equal Rise/Fall
C_{IN}/C_{REF}	
Inverter	1
NOR	$\frac{3k+1}{4}$
NAND	$\frac{3+k}{4}$
Overdrive	
Inverter	
HL	1
LH	1
NOR	
HL	1
LH	1
NAND	
HL	1
LH	1
t_{PROP}/t_{REF}	$\sum_{k=1}^{n} F_{I(k+1)}$

Equal rise-fall gates, no overdrive

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF, R_{PDREF}=2.5K

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)



F_{I2}=10.25

F_{I3}=4.25

F_{I4}=4.25

F_{I5}=1.25

F_{I6}=12.5

$$t_{PROP} = t_{REF} \sum_{k=1}^{5} F_{I_{k+1}}$$

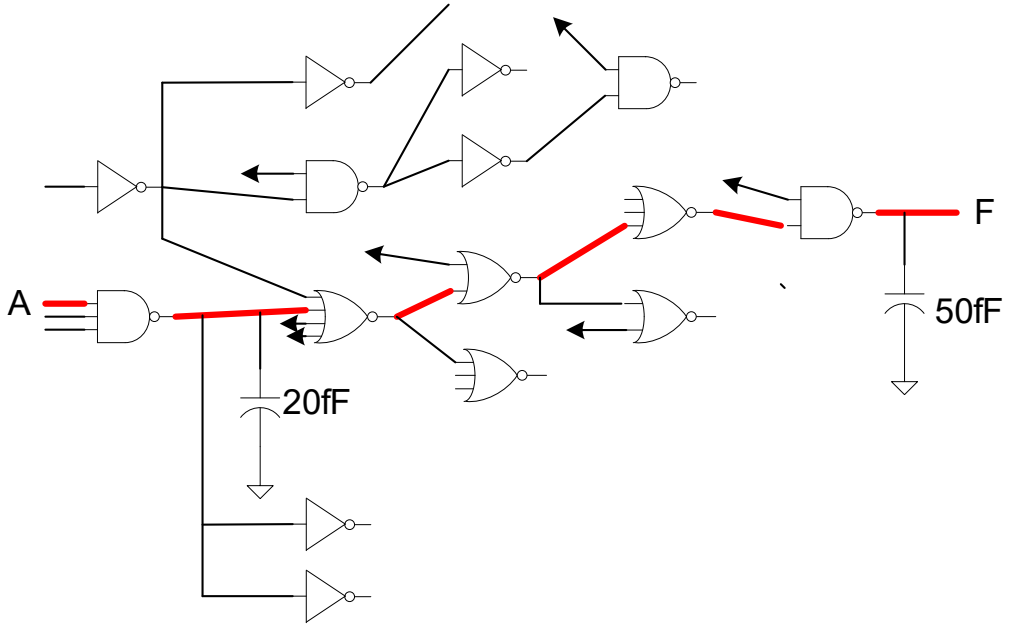
$$t_{PROP} = t_{REF} (10.25 + 4.25 + 4.25 + 1.25 + 12.5)$$

$$t_{PROP} = 32.5 t_{REF}$$

Equal rise-fall gates, no overdrive

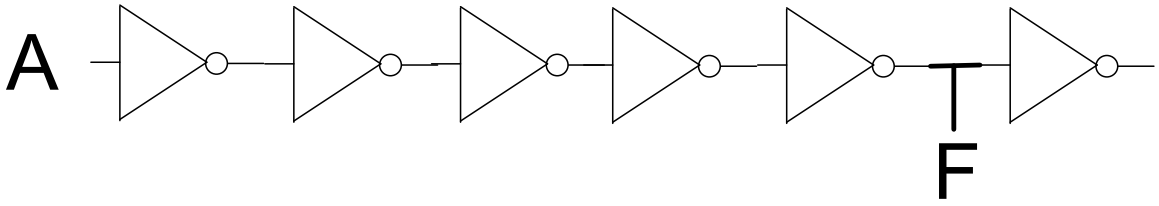
In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)



$t_{PROP}=32.5t_{REF}$

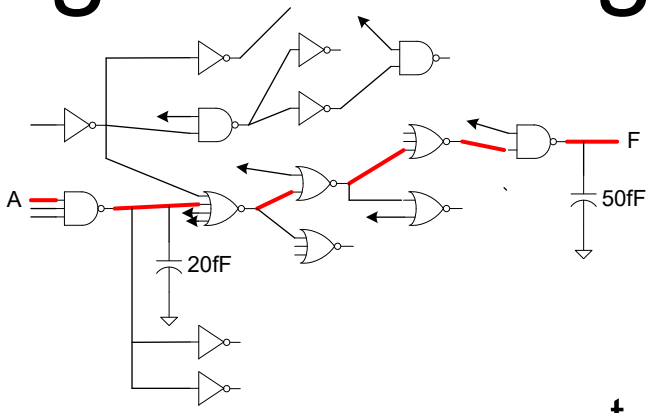
How does this propagation delay compare to that required for a propagation of a signal through 5-levels of logic with only reference inverters (load is a ref inverter instead of 50fF as well)?



$t_{PROP}=5t_{REF}$

Loading can have a dramatic effect on propagation delay

Propagation Delay in Multiple-Levels of Logic with Stage Loading



- **Equal rise/fall (no overdrive)**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

- **Equal rise/fall with overdrive**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$$

- **Minimum Sized**

$$t_{PROP} = \ ?$$

- **Asymmetric Overdrive**

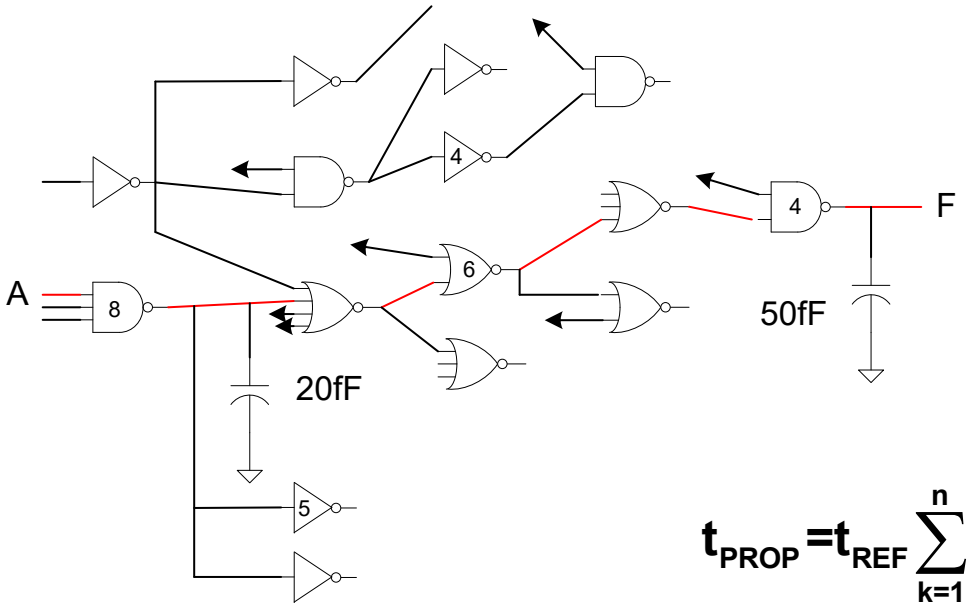
$$t_{PROP} = t_{REF} \bullet \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

- **Combination of equal rise/fall, minimum size and overdrive**

$$t_{PROP} = \ ?$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Equal rise-fall gates, with overdrive



$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I_{k+1}}}{OD_k}$$

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)

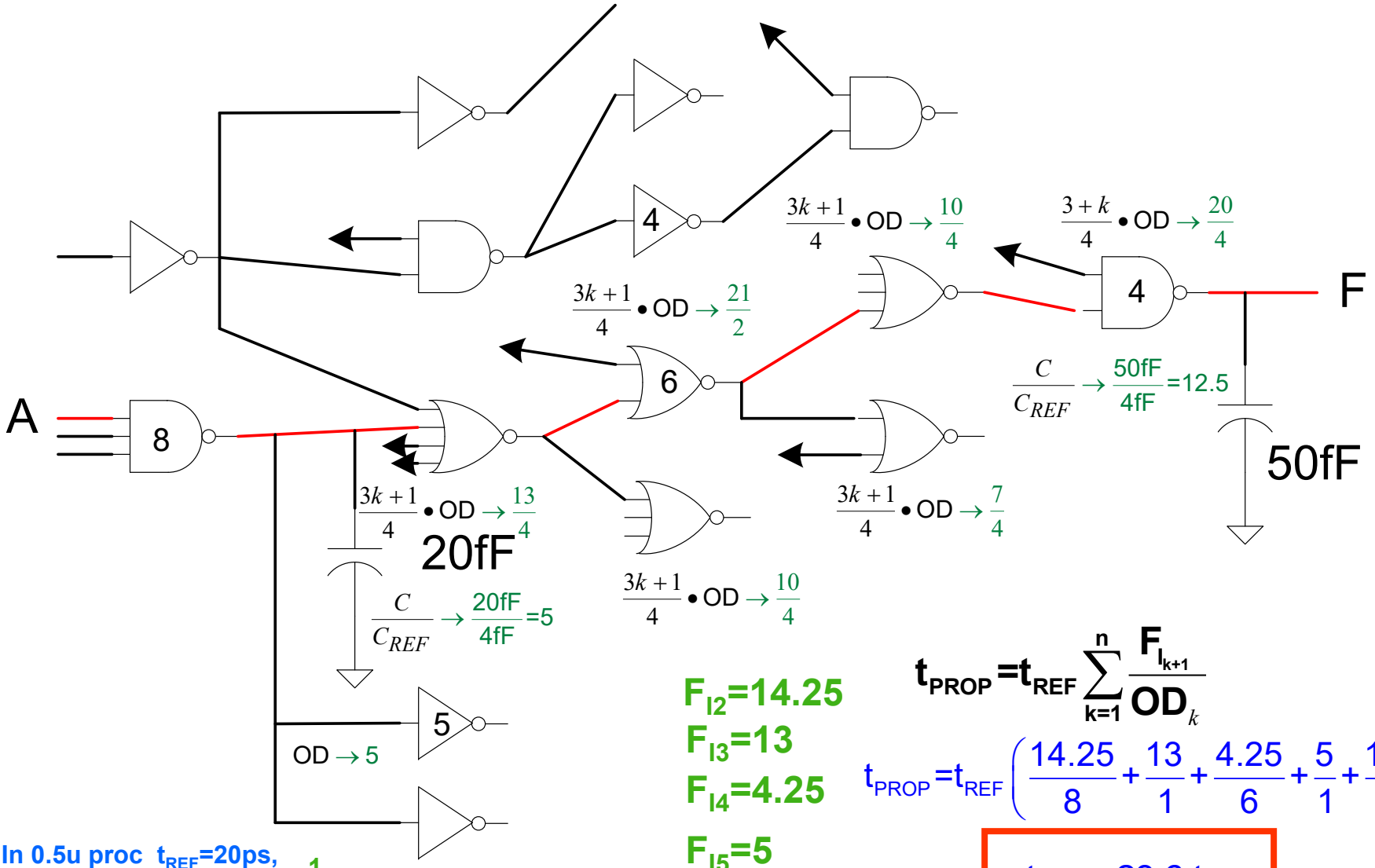
$$\frac{t_{PROP}}{t_{REF}} = \sum_{k=1}^{n} \frac{F_{I_{k+1}}}{OD_k}$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Equal rise-fall gates, with overdrive

	Equal Rise/Fall	Equal Rise/Fall (with OD)
C_{IN}/C_{REF}		
Inverter	1	OD
NOR	$\frac{3k+1}{4}$	$\frac{3k+1}{4} \bullet OD$
NAND	$\frac{3+k}{4}$	$\frac{3+k}{4} \bullet OD$
Overdrive		
Inverter		
HL	1	OD
LH	1	OD
NOR		
HL	1	OD
LH	1	OD
NAND		
HL	1	OD
LH	1	OD
t_{PROP}/t_{REF}	$\sum_{k=1}^{n} F_{I(k+1)}$	$\sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$

Equal rise-fall gates, with overdrive



$F_{I2}=14.25$

$F_{I3}=13$

$F_{I4}=4.25$

$F_{I5}=5$

$F_{I6}=12.5$

$$t_{PROP}=t_{REF}\sum_{k=1}^{n}\frac{F_{I_{k+1}}}{OD_k}$$

$$t_{PROP}=t_{REF}\left(\frac{14.25}{8}+\frac{13}{1}+\frac{4.25}{6}+\frac{5}{1}+\frac{12.5}{4}\right)$$

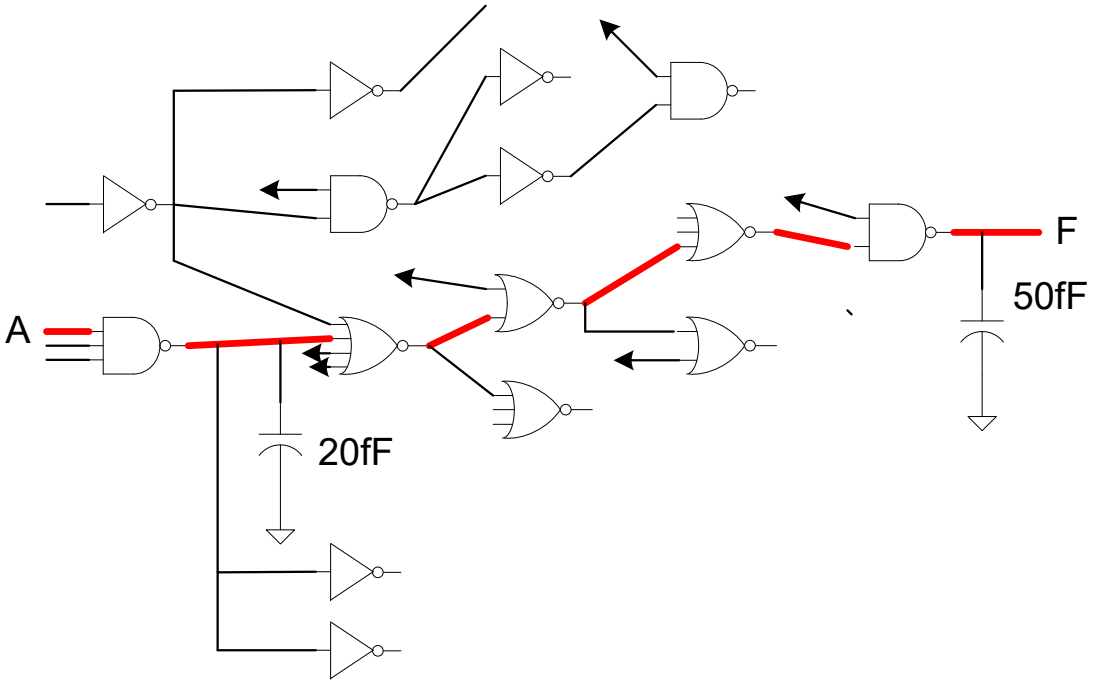
$$t_{PROP}=23.6\ t_{REF}$$

In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Minimum-sized gates

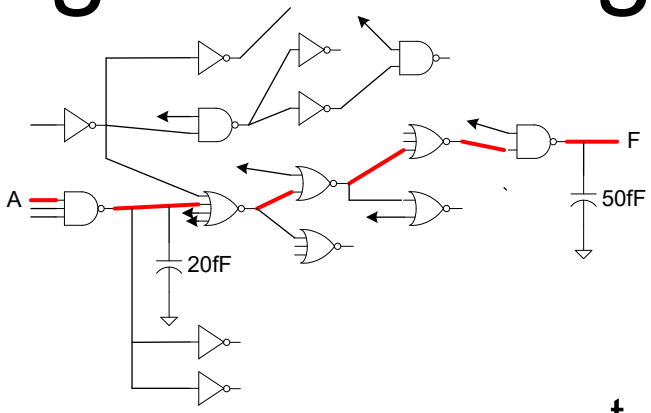


In 0.5u proc t_{REF}=20ps, C_{REF}=4fF,R_{PDREF}=2.5K

$$t_{PROP} = t_{REF} \bullet ?$$

(Note: This C_{OX} is somewhat larger than that in the 0.5u ON process)

Propagation Delay in Multiple-Levels of Logic with Stage Loading



- **Equal rise/fall (no overdrive)**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} FI_{(k+1)}$$

- **Equal rise/fall with overdrive**

$$t_{PROP} = t_{REF} \sum_{k=1}^{n} \frac{F_{I(k+1)}}{OD_k}$$

- **Minimum Sized**

$$t_{PROP} = \ ?$$

- **Asymmetric Overdrive**

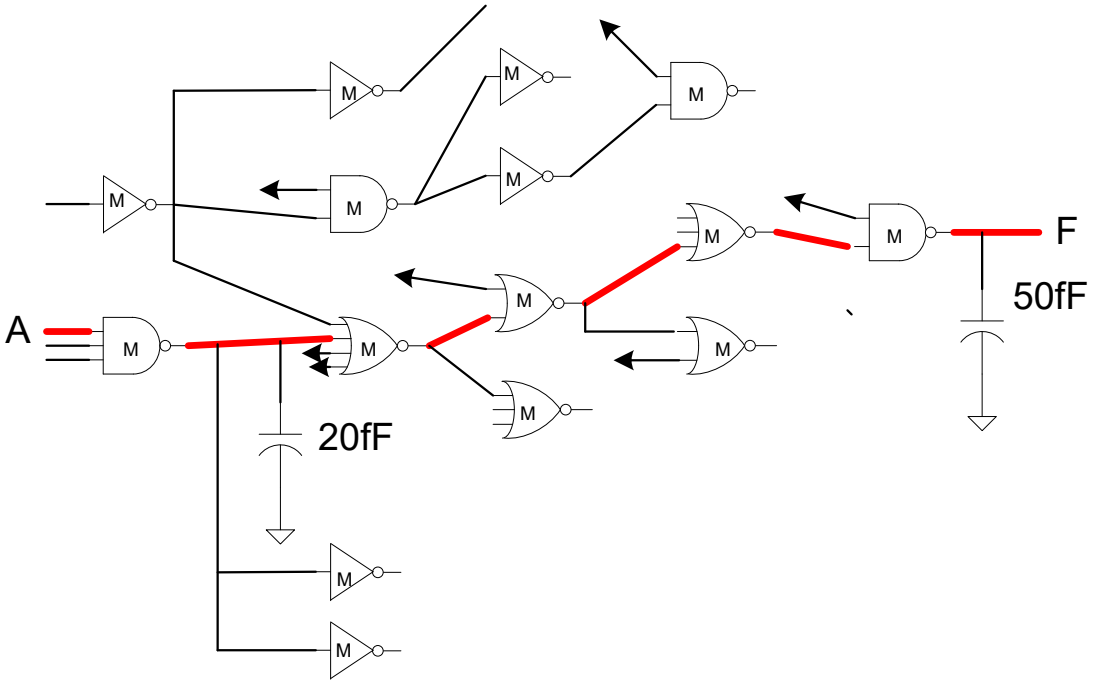
$$t_{PROP} = t_{REF} \bullet \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

- **Combination of equal rise/fall, minimum size and overdrive**

$$t_{PROP} = \ ?$$

Propagation Delay in Multiple-Levels of Logic with Stage Loading

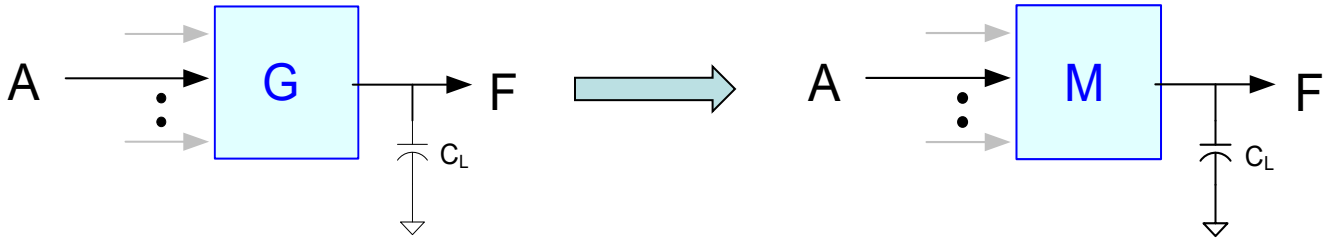
Minimum-sized gates



$t_{PROP} = t_{REF} \bullet ?$

Observe that a minimum-sized gate is simply a gate with asymmetric overdrive

Propagation Delay with Minimum-Sized Gates



Recall propagation delay for asymmetric overdrive:

$$t_{PROP} = t_{REF} \bullet \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

Thus for minimum-sized devices:

$$\frac{t_{PROP}}{t_{REF}} = \left(\frac{1}{2} \sum_{k=1}^{n} F_{I(k+1)} \left(\frac{1}{OD_{HLk}} + \frac{1}{OD_{LHk}} \right) \right)$$

- **Still need OD_{HL} and OD_{LH} for minimum-sized gates**
- **Still need FI for minimum-sized gates**

Stay Safe and Stay Healthy !

End of Lecture 41